respect to volume) injection of cold drops into the vapor in the case of an emergency pressure increase.

In the absence of phase transitions the vapor pressure also decreases when $x_1 < 1$ and $T_{10} > T_{20}$, but just because of its cooling due to thermal conductivity.

The solution for the conditions III in the case of $x_1 = 0.1$ ($\alpha_2 = 0.8 \cdot 10^{-2}$) is presented in Figs. 5 and 6. Evaporation of the particle occurs in the system. The temperature curves 1-5 in Fig. 5 correspond to the instants of time $\tau = 0.01, 2, 15, 50$, and ∞. The pressure in the system increases from $p_0 = 1$ to $p = 1.03$ bar and then decreases to $p = 0.98$ bar. This decrease in the pressure is associated with the fact that phase transition is practically discontinued but heat exchange still occurs (see Fig. 6). We note that when $x_1 < 1$ condensation is not always replaced by evaporation under conditions I.

The particle size varied little in all the alternatives discussed. In the case $x_1 = 1$ (infinite volume of vapor) this circumstance is associated with the fact that the calculations were performed prior to the emergence into quasi-time-independent conditions. One can use the time-independent solution (2.1) to describe the subsequent behavior of the system, as has already been pointed out.

In the "cell" formulation the small variation of the radius prior to the instant of establishment of equilibrium is due to the fact that alternatives with a small vapor mass content in the cell, $x_1 = 0.1$, were discussed.

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DYNAMICS OF A CYLINDRICAL CAVITY IN A COMPRESSIBLE LIQUID

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UDC 532.5.013.2+534.222.2

The equation of one-dimensional pulsation of a cylindrical cavity in a compressible liquid was derived in [1, 2] within the framework of the approximate theory of Kirkwood—Bethe [3], which is based on the approximation by the function $G = r^{1/2}\Omega$ of an invariant propagating along a characteristic at a velocity $c + u$, where $\Omega = \omega + u^2/2$ is the kinetic enthalpy, $\omega = \int dp/\rho$ is the enthalpy, u is the velocity of a fluid particle, r is the coordinate, and c is the local speed of sound.

In the derivation of this equation

$$\frac{\partial}{\partial t}[r^{1/2}(\omega + u^2/2)] = -(c+u)\frac{\partial}{\partial r}[r^{1/2}(\omega + u^2/2)] \tag{1}$$

the condition for G was used, as well as the continuity and momentum conservation equations, on the basis of which the replacement of partial derivatives by total ones was made in (1) [2]. The pulsation equation of the cavity is derived in the following form (we set $r = R$, $u = dR/dt$):

$$\begin{gathered} R[1-(dR/dt)/c]d^2R/dt^2 + (3/4)(dR/dt)^2[1-(dR/dt)/3c] = \\ = \omega[1+(dR/dt)/c]/2 + R(d\omega/dt)[1-(dR/dt)/c]/c, \end{gathered} \tag{2}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 102-106, July-August, 1977. Original article submitted May 7, 1976.

where R is the cavity radius and t is the time. The value of the enthalpy ω on the liquid side of the cavity wall is determined on the basis of the Tait equation [3] in the form

$$\omega = \frac{nB}{(n-1)\rho_0}\left[\left(1+\frac{p(R)-p_\infty}{B}\right)^{1-1/n}-1\right],$$

where B = 3050 atm and n = 7.15 are constants, ρ_0 is the fluid density, p_∞ is the pressure at infinity, p(R) is the pressure in the cavity, and the local speed of sound c is determined in the form

$$c = c_0\left(1+\frac{p(R)-p_\infty}{B}\right)^{(n-1)/2n},$$

where c_0 is the speed of sound in an undisturbed liquid.

This paper is devoted to the experimental investigation of the pulsation parameters of a cylindrical cavity and to an investigation of the characteristics of the Rich–Ginnel approximation [3], which is based on neglecting the term $\Phi/4r^2$ ($\Phi = r^{1/2}\varphi$ and φ is the velocity potential) in the wave equation, which permits (with the use of the Kirkwood–Bethe model) obtaining Eq. (1).

An analysis of the Rich–Ginnel assumption carried out for the case of an incompressible liquid (it does not seem possible to make similar estimates for a compressible liquid) has shown that it allows a definite arbitrariness in the choice of the numerical coefficient in front of $(dR/dt)^2$ in (2). Actually, one can use the following ways to construct an approximate equation of the pulsation of a cylindrical cavity in an incompressible liquid. The first way assumes a limiting transition ($c \to \infty$) in (2), whereby an equation of the form

$$R\,(d^2R/dt^2) + \frac{3}{4}\,(dR/dt)^2 = \omega/2 \tag{3}$$

is obtained. The second way consists of the fact that within the framework of the adopted approximation, in which the velocity potential is defined in the form $\varphi \approx 2R^{3/2}(dR/dt)/r^{1/2}$, expressions are found for the kinetic energy of the liquid and the potential energy of the gas, and, on the basis of the energy conservation law, the first integral of the equation of motion is written

$$2\rho_0\left[R^2(dR/dt)^2 - R_0^2(dR/dt)_0^2\right] = p_0R_0^2\,[1-(R/R_0)^{2-2\gamma}]/(\gamma-1) + p_\infty\left(R_0^2-R^2\right),$$

where p_0 is the initial pressure in the cavity. Differentiating this equation with respect to R, we obtain the cavity pulsation equation in the form

$$R(d^2R/dt^2) + (dR/dt)^2 = \omega/2. \tag{4}$$

The third way consists of the fact that the expression for the potential cited above is substituted into the Cauchy–Lagrange integral, which takes the form

$$R\,(d^2R/dt^2) + \frac{5}{4}\,(dR/dt)^2 = \omega/2 \tag{5}$$

on the cavity wall (r = R).

Thus, in the case of cylindrical symmetry the constraints imposed on the wave equation result in some indeterminacy of the numerical coefficient in front of $(dR/dt)^2$, whose values lie in the range 0.75–1.25. The fact that the form of all three equations (3)-(5) is identical and they agree, for example, in the case of large accelerations (initial stage of an explosion), when it is possible to neglect the square of the velocity, is a positive factor. It is possible to refine the value of this coefficient (let us denote it as β) from a comparison

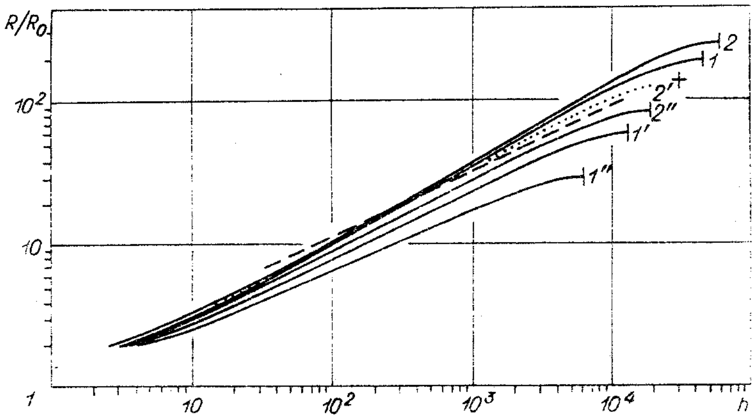

Fig. 1

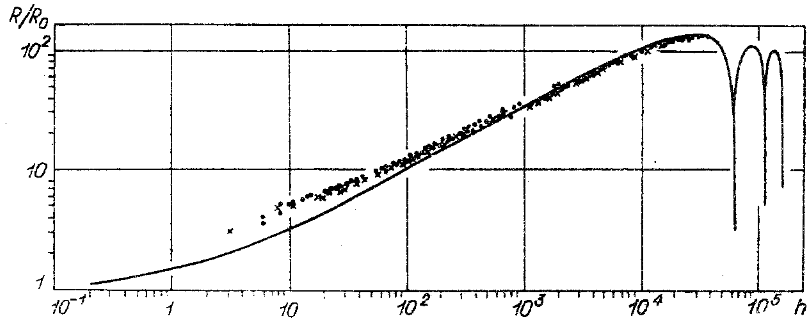

Fig. 2

of the calculated and experimental data of the pulsation of a cavity with the detonation products affiliated with the underwater explosion of cylindrical charges.

A numerical calculation of Eq. (2) for three values of β and experimental investigations for the case of an explosion of nonstandard cylindrical charges made of hexogen with a diameter d = 0.65 and 1.65 mm with a copper casing (the charge density $\rho_* = 1.55$ g/cm³ and the detonation rate D = 7.7 km/sec are determined experimentally, and the ratio of the charge length to its radius is on the order of 10^3) have been performed with the goal of determining the parameters of an explosion cavity and checking Eq. (2).

Since the comparison with the experimental data was carried out for charges of the indicated type, only the case of an "instantaneous" explosion was considered in the calculation. The initial parameters of the problem (density and speed of sound in the detonation products and the liquid, pressure at the contact explosion and its velocity) were determined from the decay condition of an arbitrary explosion. The calculation was carried out for two types of isentropes: $\gamma = 3 = \text{const}$ and a variable γ taken from [4] for the case of an "instantaneous" explosion (the density of the explosion products ρ is equal to the density of the charge, and the internal energy is equal to the heat of the explosion). Tabular data [4] are approximated in the following way:

$$
\begin{array}{ll}
0.625 \leqslant \rho^{-1} \leqslant 1.66, & p \sim \rho^{-2.78}, \\
1.66 \leqslant \rho^{-1} \leqslant 2.51, & p \sim \rho^{-2.14}, \\
2.51 \leqslant \rho^{-1} \leqslant 5.0, & p \sim \rho^{-1.73}, \\
5.0 \leqslant \rho^{-1} \leqslant 20.0, & p \sim \rho^{-1.36}, \\
\rho^{-1} > 20.0, & p \sim \rho^{-1.26}.
\end{array}
$$

Here the tabular data [4] for $\rho_* = 1.6$ g/cm³, which are closest to the experimental values, are used. At the same time a value for the pressure of $p = 1.295 \cdot 10^5$ atm corresponds to the value $\rho^{-1} = 0.625$.

The results of the calculation of the expansion of the cavity [$y(h) = R/R_0$] are presented in Fig. 1 in the scale of the charge radius R_0 as a function of the time $h = tc_0/R_0$: curves 1, 1', and 1" (for $\beta = 0.75$, 1, and 1.25, respectively) are obtained in the case $\gamma = 3$, and curves 2, 2', and 2" are for variable γ; the slope of the experimental curve $y(h)$ is denoted by a dashed line, and the experimental value of the position of the maximum y^0 is denoted by the cross. The relation 2' is illustrated especially by dots in order to distinguish the curve which is closest to the experimental data. Each relation is limited on the right by a vertical line which determines the instant of cessation of the maximally expanded cavity.

As one should have expected, the cavity parameters calculated for the case of a variable γ turned out to be nearest to the actual ones. It is evident from the graphs presented that the slope of the curves $y(h)$ decreases as the coefficient β increases: It amounts to 0.55 for the curves 2 in the case of $\beta = 0.75$, 0.5 for $\beta = 1$, and 0.49 for $\beta = 1.25$. The slope of the experimental curve is 0.45.

The same relation 2', whose plot is continued and includes three pulsations of the cavity, is presented in Fig. 2. Here the experimental data are plotted for comparison: The crosses are for a charge with diameter $d = 1.65$ mm, and the dots are for $d = 0.65$ mm. The agreement of calculation and experiment in the region $y \geqslant 10$ can be considered satisfactory.

Let us cite (in relative and absolute quantities) the principal characteristics of the pulsation of a cylindrical cavity obtained from the experimental data (denoted by an asterisk below) and from the calculation in the following:

$$\begin{gathered}
y_* = 1.5h_*^{0.45} \text{ for } 30 \leqslant h_* \leqslant 10^4, \\
R_* = 321\, R_0^{0.55} t_*^{0.45} \text{ cm} \quad \text{for } 2\cdot 10^{-4} R_0 \leqslant t_* \leqslant 6.67 \cdot 10^{-2} R_0 \mathrm{c}, \\
y_{*,1}^0 = 135\,(R_{*,1}^0 = 135 R_0), \quad y_1^0 = 141\,(R_1^0 = 141 R_0), \\
h_{*,1}^0 = 3 \cdot 10^4 (t_{*,1}^0 = 0.2 R_0 \mathrm{c}), \quad h_1^0 = 3 \cdot 10^4 (t_1^0 = 0.2 R_0 \mathrm{c}), \\
E_{*,1} = 0.22\, E_0,\ E_1 = 0.218 E_0, \quad E_2 = 0.14\, E_0,\ E_3 = 0.11\, E_0, \\
T_{*,1} = T_1 = 0.4 R_0 \mathrm{c},\ \ T_2 = 0.33 R_0 \mathrm{c},\ \ T_3 = 0.3 R_0 \mathrm{c},
\end{gathered}$$

where the subscripts 1, 2, and 3 denote the number of the pulsation cycle; the maximum values of the parameters are denoted by a superscript 0; R_0 is in cm; E_0 is the heat of explosion of the explosives per unit length; E is the energy remaining in the cavity with the detonation products after the expansion ($E_{*,1}$ is calculated over the entire maximum of the cavity volume with the detonation products); and T is the pulsation period.

The experimental and calculated values obtained for E offer the possibility of writing expressions for the pulsation periods of a cylindrical cavity in a general form in terms of the initial energy of the explosives E_0

$$T_i \simeq 1.635\,(\rho_0 \alpha_i E_0)^{1/2} p_\infty^{-1},$$

where $\alpha_i = 0.218$, 0.14, and 0.11 is the fraction of the energy of the explosives which is expended in the radial motion of liquid flow in the process of the first, second, and third pulsations.

Comparison of the empirical and calculated expressions obtained in this paper for the cavity pulsation parameters and the energy distribution among the detonation products and the shock wave in the case of cylindrical symmetry of an underwater explosion confirms the reality of Eq. (2) and permits concluding the advisability of using the coefficient value $\beta = 1$ instead of 0.75 in front of $(dR/dt)^2$.

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EXPANSION OF GAS CAVITY IN BRITTLE ROCK WITH A VIEW TO DILATATION PROPERTIES OF SOIL

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UDC 539.374

The experimental data of [1, 2] on blasts in rocks indicate that the mass velocity of the rock behind the front of the shock wave, comminuted by that shock wave, is described by the relation

$$v \sim r^{-n}, \qquad n = 1.5 - 1.8.$$

The explanation of such a relationship may be connected with the effect of dilatation in the comminuted rock, the effect consisting of the dependence of the specific volume on the plastic shear deformations [3].

The equation of continuity and the relationships imposing kinematic limitations on the velocity components [1, 3]

$$d\rho/\rho dt + \operatorname{div} v = 0, \quad (1)$$
$$I_1 - 2\Lambda\sqrt{I_2} = 0,$$

form a closed system for determining the velocity and density of the soil behind the front of the shock wave. Here ρ, v, I_1, and $I_2^{\frac{1}{2}}$ are the density, velocity, first and second invariants (deviator part) of the tensor of the deformation rate, and Λ is the dilatation rate.

The solution of the system of equations (1) in the spherical-symmetrical case with Λ = const leads to the following dependence of the velocity and density on the coordinates and the time:

$$v(r, t) = \lambda(t)\, r^{n},$$
$$\rho(r, t) = \rho^{-}(r_0)(r_0/r)^{2-n}, \; n = (2 - \Lambda)/(1 + \Lambda), \quad (2)$$

where r, r_0 are the running and initial coordinates, respectively, of the particle; $\rho^{-}(r_0)$ is the density of the material at point r_0 at the instant the shock wave passes through that point; $\lambda(t) = a^n\dot{a} = v(R)\dot{R}^n$; a and R are the radii of the cavity and of the front of the shock wave at the instant of time t; v(R) is the mass velocity of the particles behind the front of the shock wave

$$v(R) = \varepsilon(R)\,\dot{R}, \qquad \varepsilon(R) = \frac{\rho_0^{-}(R) - \rho_0^{+}}{\rho_0^{-}(R)};$$

ρ_0^{+} is the density of the soil after arrival of the shock wave.

We assume that the soil behind the shock wave is a plastic medium obeying the Mises–Schleicher condition

$$\sigma_r - \sigma_\theta = k + m(\sigma_r + 2\sigma_\theta).$$

The equation of motion in Lagrange variables can be written in the form

$$r_0^2 r^{-2}\rho_0^{+}\frac{\partial v}{\partial t} = r^{-\alpha}\frac{\partial}{\partial r_0}\left[r^{\alpha}\left(\sigma_r(r) + \frac{k}{3m}\right)\right], \quad \alpha = \frac{6m}{2m+1}. \quad (3)$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 106-109, July-August, 1977. Original article submitted July 5, 1976.

